Analytical Hysteresis Model and Initial Conditions for Inrush Current Computations using FEM

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To estimate the inrush currents occurring while energising power devices under no load conditions is the main focus of our investigations. The shape as well as the peak of the current strongly depend on the unknown magnetic history of the iron core. Hence an in-depht study of initial conditions facilitates high quality numerical simulation results. Beside this a highly accurate model of the nonlinear B-H charactercis in use is absolutely essential. At first an analytical hysteresis model will be described and a method to identify the set of parameters from measured B-H values will be shown. Hereafter, some ways how to deal with initial conditions will be discussed. After the implementation of the hysteresis model into a finite element procedure the transient answer while switching a single phase power transformer has been computed.

Index Terms—**Inrush current, Magnetisation curve, Hysteresis, First Order Reversal Curve.**

I. INTRODUCTION

THE KNOWLEDGE of the arising inrush current at switching operations advantageously should be known in advance. Control system devices in power networks have to distinguish between regular switching operations and fault cases in order to initiate proper safety measures. So, reliable numerical simulations are indispensable. It is well known that at switching of inductors and tranformers under no load conditions, the magnetic flux exhaustingly may saturate the iron core resulting in extremely high current peaks. Hence, a numerically handable and accurate hysteresis model to handle such flux peaks properly is necessary.

In our work, an analytical description of the hysteresis model in use is given. With a set of five parameters only, all analytical relations can be constructed. It is important to mention that all operation points within the major B-H loop can be addressed by so called **F**irst **O**rder **R**eversal **C**urves (FORC's, the red lines in Fig. 1), which, again are analytical expressions using the same beforementioned five parameters.

Special attention has been paid to the initial conditions serving for the transient numerical procedure.

- A first attempt (a1) to start a computation will be the assumption that the whole magnetic core has been de-magnetised. For this the magnetisation for any point within the core follow the initial magnetisation curve (blue line in Fig. 1). A complete transient procedure must be performed. The procedure must be steered by a given current profile, eg. from zero current up to a nominal current and back to zero current. Then the remaining remanent magnetic field, which means the B-H relation for any point is known. With this, a consecutive inrush current simulation will give reliable solutions for the behave of the inrush current.
- To avoid a previous transient procedure, in a second attempt (a2) a single steady-state field solution may be used. In case of the knowledge of the remanent magnetic field in one field point (due to measurements) the current excitation of the steady-state computation may be tuned to reach the known field value. It is worth mentioning that any steady-state computation operates with the initial magnetisation curve at which hysteresis is neglected. This procedure costs a very few steady-state computations, only.
- In case of not knowing any remanent field in advance, some designing hints may help to find initial conditions. In general, a device under nominal conditions is designed for a maximal magnetic field density value at some critical limb (a3). This value, again can be used to find the steady-state solution. With this as initial condition, a transient procedure taking hysteresis into account must be started while decreasing the current to zero.
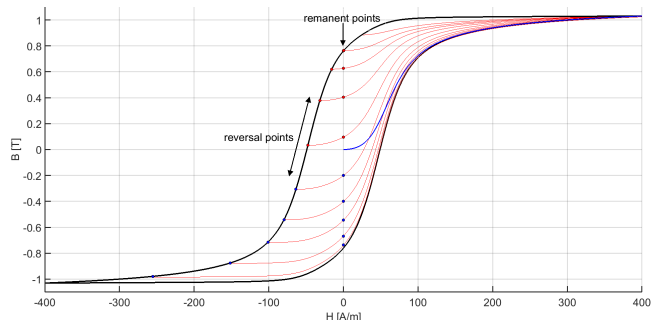

Fig. 1. *B-H* characteristic, initial magnetisation curve in blue, FORC's in red, some reversal points and remanent points indicated.

II. ANALYTICAL HYSTERESIS MODEL

$F(x)$ as the classical Langevin [1] function of its general form

$$F(x) = \coth(x) - \frac{1}{x} \tag{1}$$

serves for the description of soft magnetic materials. It is an odd function and possesses a distinctive saturation behaviour. As described in [2], after shrinking or streching and transforming the basic function (1), some combination can be found to model the initial magnetisation curve

$$
\begin{aligned}
M(H) \;=\; & M_a\, F\left(\frac{H}{H_a}\right) + \\
& + \frac{M_b}{4}\left[F\left(\frac{H+H_c}{H_b}\right) + F\left(\frac{H-H_c}{H_b}\right)\right]^2 \\
& - H_{max} \;\leq\; H \;\leq\; +H_{max}
\end{aligned}
\tag{2}
$$

as well as the FORC's by

$$
\begin{aligned}
M(H) \;=\; & M_a\, F\left(\frac{H}{H_a}\right) \pm \\
& \pm \frac{M_b}{2}\left[F\left(\frac{H+H_c}{H_b}\right) - F\left(\frac{H_r+H_c}{H_b}\right)\right] * \\
& * \left[F\left(\frac{H-H_c}{H_b}\right) - F\left(\frac{H_r-H_c}{H_b}\right)\right].
\end{aligned}
\tag{3}
$$

Herein are M_a the reversible component of the magnetisation, M_b the maximum of the magnetisation, H_a the rate to approach M_b, H_b the rate to approach saturation and H_c the coercitivity field intensity. In (3) setting for H_r the maximum values $\pm H_{max}$ one obtaines the main branches of the hysteresis loop, as worked out in [3]. From the magnetisation $M(H)$, the $B(H)$-functions may be expressed by using

$$
B(H) \;=\; \mu_0\left(M(H) + H\right).
\tag{4}
$$

All five parameters can be found with a set of n (B_{m_i}, H_{m_i}) measured couples of an initial curve by applying a standard evolution strategy (eg. [4]) to minimise the error

$$
\sum_{i=1}^{n} \left[B(H_{m_i}) - B_{m_i}\right]^2 \rightarrow \text{minimise}
\tag{5}
$$

between the measurements and the analytical solution $B(H_{m_i})$ obtained from (2) and (4).
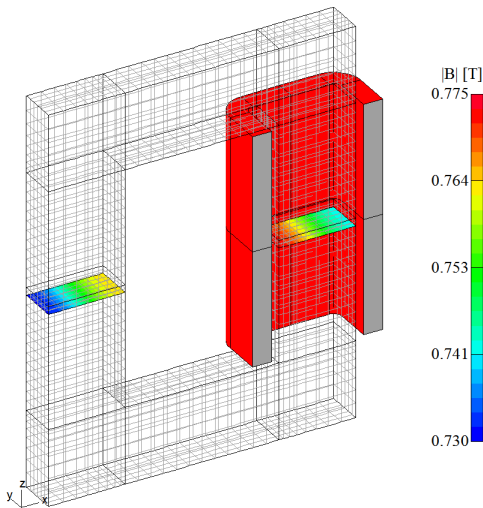
Fig. 2. Half of the structure of the test configuration, initial field distribution of B_{rem}, obtained by a steady-state computation (a2).

III. NUMERICAL INVESTIAGTIONS

A test configuration consists of a rectangular iron core and a racetrack shaped coil, placed around one limb of the core. In Fig. 2 along two arbitrarily selected cross sections the B_{rem} distribution is shown. The distributions have been found by following the attempt a2.

For computing the initial conditions, the coil always is excited by a current. In contrast to this the coil will be driven by a sinusoidal voltage to obtain the inrush current behaviour. The latter enforces to solve the underlying network equations. With the impressed voltage $u_0(t)$

$$
u_0(t) = \sqrt{2}\,U_0 \sin(\omega t + \varphi)
\tag{6}
$$

Faraday's law becomes

$$
R\,i(t) - u_0(t) = -\frac{d\Phi}{dt}.
\tag{7}
$$

R represents the ohmic resistor of the windings and $i(t)$ is the inrush current we are looking for. The change in time of the flux linkage $\frac{d\Phi}{dt}$ stems from the FEM-simulation. Due to the nonlinearity of the FORC's, at each time instant an iterative computation loop has to passed through in order to find the proper current value to satisfy (7). Fig. 3 shows two solutions
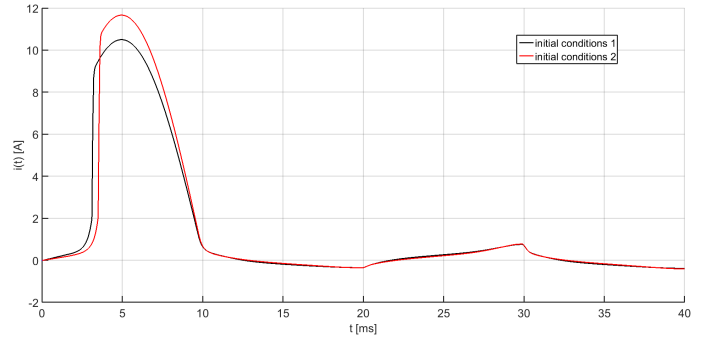
Fig. 3. Lapse of the inrush current $i(t)$ for two different initial conditions.

for the inrush current. Corresponding to attempt a2, a steady-state solution with $B_{rem} = 0.775$ T at a specified point results in the black line (initial conditions 1). On the other hand, the red line is predicated on a constant magnetic field $B_{rem} = 0.775$ distribution all over the core (initial conditions 2). A distinctive increase in the peak value must be observed. Hence, a carefull choice of proper initial conditions must be kept in mind.

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